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POSSIBILITY, EXISTENCE, AND AN ONTOLOGICAL ARGUMENT

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In 'Why Is There Something and Not Nothing?' (*Analysis* **26**, 6 (June 1966), 177–181), Fred Sommers presents an argument intended to show "that if something is possible, something is actual" (178). The argument proceeds by way of *reductio ad absurdem* from three assumptions and a definition; but since the definition is not wholly unproblematic I shall treat it also as simply an assumption on a par with the other three.

Sommers' three assumptions are

(A1) "Something is possible" (177).

(A2) "Whatever is not a categorially possible thing is not a possible thing" (*ibid.*).

and

(A3) There is nothing.

And, taking $\ulcorner D \urcorner$ to abbreviate a monadic general term and $\ulcorner \bar{D} \urcorner$ its logical contrary, Sommers' fourth assumption, which acts as his definition of categorial possibility, is

(A4) "*D*-things are categorially impossible, if and only if there is nothing that is *D* and nothing that is $\bar{D}$" (*ibid.*).

Sommers argues that these four, taken together, generate an inconsistency. (A3) is the most natural premise to reject, and so Sommers represents his argument as showing that from the assumption that something is possible, together with the unproblematic (A2) and (A4), we can derive the denial of (A3), that is, that something exists. Intuitively, it is no less straightforward to view Sommers' argument as going directly from (A1), (A2) and (A4) to the denial of (A3): If *D*-things are possible [(A1)], then by (A2) *D*-things are categorially possible; but by (A4), *D*-things are categorially possible only

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if they are either D or non D. It follows that either D-things or non$D$-things exist, and hence that something exists.

In several more recent articles,[1] Sommers has urged that a 'nonquantificational logic' based on the idea that all sentences basically have subject-predicate form is preferable to quantificational logic, which, according to Sommers, wrongly requires that all predication be singular predication (see especially 'On a Fregean Dogma', *passim*). This might suggest that to test Sommers' surprising argument about possibility and existence for validity, one should use a nonquantificational calculus of terms of the sort Sommers propounds. In what follows, I shall not explore this possibility, but shall instead try to find a rigorous quantificational formulation of Sommers' argument that is at once both valid and non-question-begging. If, as I shall argue, there is no such formulation, we may conclude that either Sommers' argument does not show what it purports to, or it does so, if at all, only relative to some nonstandard system of inference.

If Sommers' argument is to make use of (A4), it seems that it must proceed by way of talking about D-things, for variable $\ulcorner D \urcorner$, rather than by talking, without qualification, about individuals. We can readily accomodate this feature of the argument by making use of second-order quantification. But the use we make will be entirely benign for those with Quinean scruples; the second-order quantifiers will in every case be initial with their scope extending to the end of the formula. And since Sommers deliberately avoids defining "possibility in any general way" (177), save in asserting (A2) and in giving an example of something that is both categorially possible and "impossible in some other way" (*ibid.*), we shall use the operator '$\Diamond$', as ordinarily understood, to express possibility *tout court*. And, again in an unproblematic way, we shall use a superpositioned bar to construct the predicate logically contrary to a particular predicate.

Sommers moves directly from 'There is nothing', to its being "true for every predicate term P, that nothing is P ... [and] also true that there is nothing that fails to be $\bar{P}$" (177–178). For simplicity, then, we may begin by formulating (A3) as

(A3.1) $(P)\,(x)\,(-Px \wedge -\bar{P}x)$.

We may then construct a valid derivation of an inconsistency by representing the remaining assumptions as

(A1.1) $(\exists P)(\exists x)\Diamond Px$
(A2.1) $(P)(x)(-\Delta Px \supset -\Diamond Px)$

(using 'Δ' to abbreviate 'it is categorially possible that'), and

(A4.1)	$(P)[(x)(-Px \wedge -\bar{P}x) \equiv (x)-\Delta Px]$	
(5)	$\Diamond Qx$	(A1.1), E.I. twice
(6)	$(x)(-Qx \wedge -\bar{Q}x) \equiv (x)-\Delta Qx$	(A4.1), U.I.
(7)	$(x)(-Qx \wedge -\bar{Q}x)$	(A3.1), U.I.
(8)	$(x)-\Delta Qx$	(6) (7), T.F.
(9)	$-\Delta Qx$	(8), U.I.
(10)	$-\Delta Qx \supset -\Diamond Qx$	(A2.1), U.I. twice
(11)	$-\Diamond Qx$	(9) (10), T.F.
(12)	$p \wedge -p$	(5) (11), T.F.

This reconstruction of Sommers' argument, however, is not satisfactory. For the formulation of (A1) by (A1.1) is transparently question-begging in that it asserts not that it is possible that something exists, but rather that something exists that is possibly P. It can be no surprise that from (A1.1) one can prove that something exists; (A1.1) asserts that. The only need for the additional premises (A2.1) and (A4.1) in the above proof is due to our having followed Sommers in formulating 'Something exists' as (equivalent to) '$(\exists P)(\exists x)(Px \vee Px)$', rather than, for example, as '$(\exists x)(x = x)$'. Avoiding the question-begging (A1.1) is a straightforward matter, for it is possible, and perhaps more natural, to formulate (A1) by

(A1.2) $(\exists P)\Diamond(\exists x)Px$.

But if the remaining formulations remain unchanged, using (A1.2) avoids a question-begging premise only at the expense of losing the ability to derive an inconsistency. For (A1.2), unlike (A1.1), does not yield step (5) in the above proof, at least not unless we take '$\Diamond(\exists x)Px$' to imply '$(\exists x)\Diamond Px$'.[2] But this implication is no less question-begging, in a proof that if something is possible then something exists, than the initial formulation of 'Something is possible' by (A1.1). In fact, this implication is equivalent to the validity of the familiar Barcan formula of quantified modal logic,

$$(x)\Box Px \supset \Box(x)Px$$

And as Kripke has shown (Saul A. Kripke, 'Semantical Considerations on

Modal Logic', *Acta Philosophica Fennica*, Fasc. **XVI** (1963), 87–88), this formula is valid only if we assume, counterintuitively, that any individual that exists in any world that is possible relative to the actual world also exists in the actual world. (See G. E. Hughes and M. J. Cresswell, *An Introduction to Modal Logic* (London: Methuen and Co.Ltd., 1968, Ch. 10, especially 177–178), for further discussion of existence assumptions required for the Barcan formula to be valid.)

It is not difficult, however, to so revise the above formulations of Sommers' remaining assumptions that an inconsistency can once again be validly derived, using (A1.2). Leaving (A3.1 unmodified since this formulation is Sommers' own, (A4.1) can be changed so that the modal operator in the right-hand side of the biconditional governs the first-order quantifier, as in (A1.2):

(A4.2) $(P)[(x)(-Px \wedge -\bar{P}x) \equiv -\Delta(\exists x)Px]$

And both the antecedent and the consequent of (A2.1) can also be changed, in an exactly parallel fashion, confining the scope of the individual quantification to the separate components:

(A2.2) $(P)(-\Delta(\exists x)Px \supset -\Diamond(\exists x)Px)$

(This formulation was suggested to me by Sommers, in private communication.) A derivation of inconsistency can now proceed:

(5′)	$\Diamond(\exists x)Qx$	(A1.2), E.I.
(6′)	$(x)(-Qx \wedge -\bar{Q}x) \equiv -\Delta(\exists x)Qx$	(A4.2), U.I.
(7)	$(x)(-Qx \wedge -\bar{Q}x)$	(A3.1), U.I.
(8′)	$-\Delta(\exists x)Qx$	(6′) (7), T.F.
(9′)	$-\Delta(\exists x)Qx \supset -\Diamond(\exists x)Qx$	(A2.2), U.I.
(10′)	$-\Diamond(\exists x)Qx$	(8′) (9′), T.F.
(11′)	$p \wedge -p$	(5′) (10′), T.F.

An initial objection to this proof is that (A2.2) may not appear to capture accurately Sommers' assumption (A2), "that whatever is not a categorially possible thing is not a possible thing." For the word 'whatever', here, suggests a first-order universal quantifier binding the individual variables in both antecedent and consequent of the conditional, as in (A2.1). (A2.2) seems to say not that whatever is not categorially possible is not possible, but rather that whatever sort of thing is not categorially possible is not possible. But recalling the importance, for Sommers, of using predicate variables in formulating (A3) and (A4), it is not unreasonable to permit the 'whatever' of (A2) to be expressed solely by the second-order quantifier.

Taken together, however, assumptions (A2.2) and (A4.2) beg the question at issue no less than (A1.1) did in the first proof above. For (A2.2) tells us that if it is possible that something exists that is P then it is categorially possible that something exists that is P; and (A4.2) asserts that the latter is true just in case there exists something that is either P or non$P$. Like the inference from '$\Diamond(\exists x)Px$' to '$(\exists x)\Diamond Px$', the combination of (A2.2) and (A4.2) have the effect of converting the positions of the existential quantifier and the possibility operator in such a way that enables one to go from what is possible what actually exists.

Herbert Guerry ('Sommers' Ontological Proof', *Analysis* 28, 2 (December 1967), 60–61) has criticized Sommers' (A2) and (A4) on substantially similar grounds, that taken together they permit one to infer from what is contingently not the case to what is not even possible. The present argument does not take issue with this inference as such, but rather seeks to show that Sommers has, in assuming its validity, argued in a viciously circular manner. Guerry's criticism of this inference, moreover, relies on his intuitive understanding of Sommers' notion of categorial possibility, and in particular on assumptions about what things are categorically possible at particular times. Pending a clearer explication of categorial possibility than Sommers has given, however, it is not clear what he should make of Guerry's examples. And in any event, it is sufficient to show that Sommers' argument is, in his own terms, question-begging.

No combinations of the above formulations of Sommers' premises other than those used in the two proofs permit the valid derivation of an inconsistency. So no combination other than those two can yield a valid argument that if something is possible, then something exists. Sommers' text clearly requires formulating (A3) as (A3.1); and the two formulations each of (A1) and (A4) exhaust the possibilities, for changing the scopes of modal operators and quantifiers seems to be the only way to construct distinct formulations of these premises. More room exists to construct different versions of (A2), since it contains more modal operators. But formulations other than (A2.1) and (A2.2) that result from scope changes of different quantifiers either yield something equivalent to (A2.1) or (A2.2), or fail to make possible a valid derivation of inconsistency. So if Sommers' argument can be adequately represented in quantificational notation at all,[3] and his recommendation of (A2.2) implies his belief that it can, his argument is either invalid or viciously circular.

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NOTES

[1] 'On a Fregèan Dogma', in *Problems in the Philosophy of Mathematics*, ed. by Imre Lakatos, Amsterdam, North-Holland Publishing Co., 1967, 47–81; 'Do We Need Identity?', *The Journal of Philosophy* LXVI, 15 (August 7, 1969), 499–504; and 'The Calculus of Terms', *Mind* LXXIX, 313 (January 1970), 1–39.

[2] One can, however, show that (A1.2) together with the necessity of each of (A2.1), (A3.1) and (A4.1), jointly imply an inconsistency. By a proof parallel to that in the text, one can first derive an inconsistency from (A2.1), (A3.1) and (A4.1) themselves, and '$(\exists x)Qx$' in place of (A1.1):

(5.1)	$(\exists x)Qx$	
(5.2)	Qx	(5.1), E.I.
(5)	$\Diamond Qx$	(5.2), Axiom of necessity and T.F.

Steps (6) through (12) remain exactly as is. So by conditional proof (using '(A2.1)', '(A3.1)' and '(A4.1)' to abbreviate the formulas they name)

(13)	$[(A2.1) \wedge (A3.1) \wedge (A4.1)] \supset -(\exists x)Qx$

is a theorem. The following proof then sufficies:

(14)	$\Box[[(A2.1) \wedge (A3.1) \wedge (A4.1)] \supset -(\exists x)Qx]$	(13), Necessitation
(15)	$\Box[(A2.1) \wedge (A3.1) \wedge (A4.1)] \supset \Box-(\exists x)Qx$	(14), Distributivity of '□' over '⊃'
(16)	$[\Box(A2.1) \wedge \Box(A3.1) \wedge \Box(A4.1)] \supset \Box-(\exists x)Qx$	(15), Distributivity of '□ over '∧'
(17)	$(P)[[\Box(A2.1) \wedge \Box(A3.1) \wedge \Box(A4.1)] \supset \Box-(\exists x)Px]$	(16), U.G.
(18)	$[\Box(A2.1) \wedge \Box(A3.1) \wedge \Box(4.1)] \supset (P)\Box-(\exists x)Px$	(17), Confinement of universal quantifier
(19)	$[\Box(A2.1) \wedge \Box(A3.1) \wedge \Box(A4.1)] \supset -(\exists P)\Diamond(\exists x)Px$	(18), Definitions of '(∃)' and '◇'
(20)	$[\Box(A2.1) \wedge \Box(A3.1) \wedge \Box(A4.1) \wedge -(\exists P)\Diamond(\exists x)Px] \supset (p \wedge -p)$	(19), T.F.

(I am grateful to Richard E. Grandy for pointing out the possibility of this proof.)

Not only is this proof implausible as a reconstruction of Sommers' argument, but it shows nothing whatever of use to him. If (A4.1) is acceptable as Sommers' definition of categorial possibility, and (A2.1) as a principle connecting categorial possibility with possibility *tout court*, both can reasonably be regarded as necessarily true. So, from the necessity of (A2.1) and (A4.1), along with (A1.2), one can derive

$$-\Box(P)(x)(-Px \wedge -\bar{P}x)$$

But this is equivalent to

$$\Diamond(\exists P)(\exists x)(Px \vee \bar{P}x)$$

which is weaker than (A1.2) itself, and says not that something actually exists, but again only that it is possible that something does.

[3] In 'Sommers on Empty Domains and Existence', (*Notre Dame Journal of Formal Logic* **XIII**, 3 (July 1972), 350–358), George Englebretsen has given a proof using apparatus that is not clearly translatable into quantificational notation, to try to establish that if something is possible, then something exists (357–358). Because of the nonstandard apparatus (it is hard to see how standard syntactic and semantic metatheorems could even be formulated for his apparatus), and because 'exists' construed as a general term is the only nonlogical particle to occur anywhere in Englebretsen's proof, it is difficult to determine whether he has established his result in a non-question-begging way. And this is all the more difficult to do since Englebretsen, unlike Sommers, permits 'fails to exist' taken as a predicate to be applicable to some things, and indeed to apply provably to some ('Sommers on the Predicate "Exists",' *Philosophical Studies* **26**, 5/6 (December 1974), 419, 422). A proof of this, according to Englebretsen, "is, in principle, comparable to" his proof that if something is possible, then something exists (fn. 6). In 'Sommers' Proof That Something Exists' (*Notre Dame Journal of Formal Logic* **XVI**, 2 (April 1975), 298–300), Englebretsen argues that Sommers' own proof is defective. But like Guerry, Englebretsen takes issue with that proof on the basis of a particular understanding of categorial possibility. It is far from clear that Englebretsen shares Sommers' notion of categorial possibility (299), particularly in respect to 'exists' taken as a predicate ('Sommers on the Predicate "Exists",' 419). Compare Sommers' 'Existence and Predication' (in *Logic and Ontology*, ed. by Milton K. Munitz, New York, New York University Press, 1973, 159–174), in which he argues "against the popular view that existence is syncategorematic" (174).